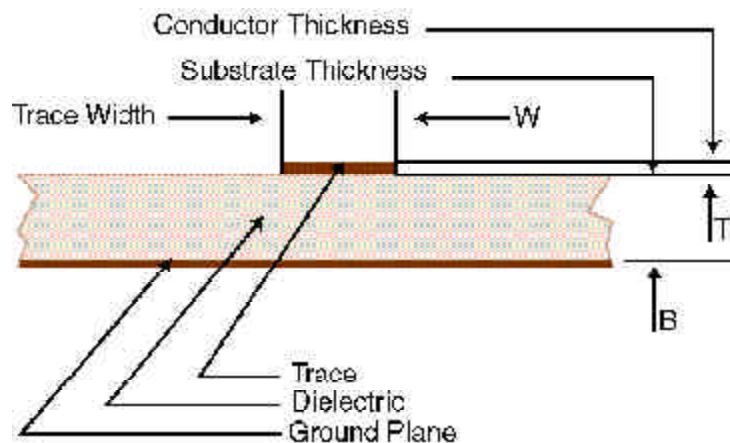


## Width and Effective Dielectric Constant Equations for Design of Microstrip Transmission Lines

As a convenience for microstrip design, line width and effective dielectric constant can be calculated by using our impedance calculation program. This free software is available for use or download on our website at <http://www.rogerscorporation.com/acm>.

Unlike the stripline configuration where the strip lies between two ground planes, transmission lines in microstrip are not entirely in the TEM mode and tend to be dispersive. That is, the effective dielectric constant and the impedance vary with the frequency of the transmitted signal. The effect is especially pronounced when the frequency is near the TEM cutoff frequency.

Several papers have been written dealing with the microstrip transmission line. The static electric field computer analysis of Bryant and Weiss (1,2) is accepted as highly accurate, but lacking in frequency effect and time-consuming in computation. The standard closed form solutions of Wheeler (3) and Schneider (4) have been improved in accuracy by Hammerstad (5) and a summary of accurate closed form equations, including the effect of frequency, have been presented by Hammerstad and Jensen (6). Improved formulas for the frequency dispersion effect were published by Kirschning and Jansen (7). These were experimentally



1. T.G. Bryant and J.A. Weiss, "Parameters of Microstrip Transmission Lines and Coupled Pairs of Microstrip Lines". IEEE Trans MTT, MTT-16 (Dec. 1968) pp 1021-27.
2. T.G. Bryant and J.A. Weiss, "MSTRIP (Parameters of Microstrip)", IEEE Trans MTT, Computer Prog. Desc. (Apr. 1971) pp 418-419.
3. H.A. Wheeler, "Transmission Line Properties of Parallel Strips Separated by a Dielectric Sheet", IEEE Trans MTT, (Mar. 1965) pp 172-185.
4. M.V. Schneider, "Microstrip Lines for Microwave Integrated Circuits", The Bell System Technical Journal, (May-June 1969) PP 1421-44.
5. E.O. Hammerstad, "Equations for Microstrip Circuit Design", Proceedings of 5th European Microwave Conference, (Sept 1-4, 1975) Hamburg, Germany, pp 268-72.
6. E. Hammerstad and O. Jensen, "Accurate Models for Microstrip Computer-Aided Design", 1980 IEEE MTT-S International Symposium Digest, (May 1980) Washington D.C. IEEE catalog #80CH1545-3MTT, pp 407-409.
7. M. Kirschning and R.H. Jansen, Electronics Letters, (18 March 1982) Vol 18, No. 6, pp 272-273.
8. S. Deibele, J.B. Bayer, IEEE Trans MTT, Vol MTT-35, No. 5, (May 1987) pp 535-538.
9. PLANAR TRANSMISSION LINE STRUCTURES, Ed, T.Itoh, 1987 IEEE Pres.

evaluated by Deibele and Bayer (8), who reported better prediction of results, especially at higher frequencies. The formulas in (6) with dispersion according to (7) were used in this revision. References 1, 6 are included in the collection of reference 9.

The following formulas can be used to generate tables:

Description of symbols:

- B = substrate thickness in millimeters  
 T = ratio of conductor thickness to substrate thickness  
 U = ratio of trace width to substrate thickness  
 F = frequency in GHz  
 $\epsilon_r$  = relative permittivity  
 $\epsilon_{\text{eff},0}$  = effective relative permittivity at 0 frequency  
 $\epsilon_{\text{eff},f}$  = effective relative permittivity at frequency F  
 $Z_0$  = characteristic impedance at 0 frequency  
 $Z_{0,f}$  = characteristic impedance at frequency F  
 $\eta_0$  = 376.73  
 P = filling factor  
 c = speed of light : 299.792 mm/ns  
 e = natural logarithm base

Get width corrected for thickness in homogeneous medium

$$U_1 = U + \frac{T \ln[1 + \frac{4e}{T} \tanh^2((6.517U)^{0.5})]}{p}$$

Get width corrected for mixed media

$$U_r = U + \frac{(U_1 - U)(1 + \frac{1}{\cosh((\epsilon_r - 1)^{0.5})})}{2}$$

Use  $U_r$  and  $\epsilon_r$  to get value for intermediate Y

$$Y = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} (1 + \frac{10}{U_r})^{-(A_u \cdot B_{er})}$$

$$A_u = 1 + \frac{\ln(\frac{U_r^4 + \frac{U_r^2}{2704}}{U_r^4 + 0.432})}{49} + \frac{\ln[(\frac{U_r}{18.1})^3 + 1]}{18.7}$$

$$B_{er} = 0.564 (\frac{\epsilon_r - 0.9}{\epsilon_r + 3})^{0.053}$$

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Publication #

Get  $Z_0$  without frequency effect

$$Z_0 = \frac{Z_{01}(U_r)}{Y^{0.5}}$$

Where  $Z_{01}$  (arg) is given by

$$Z_{01}(x) = \frac{h_0 \ln[\frac{(6 + (2p - 6)e^{-\frac{(30.666)^{0.7528}}{x}})}{x} + (\frac{4}{x^2} + 1)^{0.5}]}{2p}$$

Get effective permittivity without frequency effect

$$\epsilon_{\text{eff},0} = Y (\frac{Z_{01}(U_1)}{Z_{01}(U_r)})^2$$

The filling factor P is from Kirschning and Jansen. This was found more accurate than the simpler one in Hammerstad and Jensen according to measurements reported by Deibele and Bayer.

$$P_1 = 0.27488 + U[0.6315 + 0.525(0.0157FB + 1)^{-20}] - 0.065683e^{-8.7513U}$$

$$P_2 = 0.33622(1 - e^{-0.03442e_r})$$

$$P_3 = 0.0363e^{-4.6U} (1 - e^{-\frac{FB}{38.7}^{4.97}})$$

$$P_4 = 2.751(1 - e^{-\frac{e_r}{15.916}^8}) + 1$$

$$P = P_1 P_2 [FB(0.1844 + P_3 P_4)]^{1.5763}$$

Apply the filling factor to the Getsinger dispersion model for the effective permittivity and characteristic impedance at frequency F

$$\epsilon_{\text{eff},F} = \epsilon_r - \frac{\epsilon_r - \epsilon_{\text{eff},0}}{1 + P}$$

$$Z_{0,F} = Z_0 (\frac{\epsilon_{\text{eff},0}}{\epsilon_{\text{eff},F}})^{0.5} \frac{\epsilon_{\text{eff},F} - 1}{\epsilon_{\text{eff},0} - 1}$$